REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

(2)

AD-A232 297

2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	Reprint

4. TITLE AND SUBTITLE	5. FUNDING NUMBERS
Title shown on Reprint	DAAL03-86-K-0056
6. AUTHOR(S) Authors listed on Reprint	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) U. of Minnesota, Minneapolis, Minn. 55455	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING MONITORING AGENCY NAME(S) AND ADDRESS(ES) U. S. Army Research Office, P. O. Box 12211, Research Triangle Park, NC 27709-2211	10. SPONSORING/MONITORING AGENCY REPORT NUMBER ARO 23422.1-EG

11. SUPPLEMENTARY NOTES
The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION AVAILABILITY STATEMENT	12b. DISTRIBUTION CODE
Approved for public release; distribution unlimited.	

13. ABSTRACT (Maximum 200 words)

ABSTRACT SHOWN ON REPRINT

14. SUBJECT TERMS	15. NUMBER OF PAGES
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev 2-89)
Prescribed by ANSI Std Z39-18
298-102

AKO 23422.1-EG

Design of a Multivariable Helicopter Flight Control System for Handling Qualities Enhancement

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New handling qualities specifications are currently being developed for attack helicopters. Most unaugmented helicopters will not meet these specifications and feedback control is necessary to improve handling qualities so that safe operation close to the earth in poor weather conditions and/or at night is possible. In this paper a methodology for the direct design of helicopter flight control systems which meet handling qualities specifications is presented. This methodology uses full state feedback to place closed loop eigenvalues to achieve bandwidth specifications and to shape closed loop eigenvectors to decouple lateral and longitudinal responses to control inputs. Full state feedback requires that all state variables be known; however, only angular rates and normal acceleration are measured by sensors. Thus, a state estimator is required in the feedback loop in order to convert sensor outputs to control inputs. This estimator is designed using eigenstructure assignment so as to achieve loop transfer recovery. Design of a feedback system for use in precise hovering control for a modern attack helicopter is used to illustrate the method. Control law synthesis is accomplished using an eighth order model which includes only rigid body modes. Control law performance is evaluated using a 37th order model which includes rigid body, actuator, rotor, sensor, and flexure dynamics. It is found that a notch filter must be added to the design in order to eliminate a high frequency instability. Once this is accomplished, both the time and frequency response characteristics of the augmented helicopter are much improved compared with the unaugmented helicopter.

Notation

Scalars

i = $\sqrt{-1}$
l = arbitrary number of eigenvalues
m = number of controls
n = number of states
n_z = normal acceleration of c.g., g's
p = roll rate, rad/sec
q = pitch rate, rad/sec
r = yaw rate, rad/sec
s = Laplace operator
u = forward velocity, ft/sec
v = lateral velocity, ft/sec
w = downward velocity, ft/sec
z_i = i-th transmission zero
u_1 = collective pitch, deg
u_2 = longitudinal cyclic pitch, deg
u_3 = lateral cyclic pitch, deg
u_4 = tail rotor collective pitch, deg
$\underline{\sigma}(A)$ = minimum singular value of matrix A
$\bar{\sigma}(A)$ = maximum singular value of matrix A
θ = pitch angle, rad
φ = roll angle, rad

Vectors

u = control vector, $[u_1, u_2, u_3, u_4]^T$
u_c = command control vector from control mixer
x = state vector, $[u, v, w, p, q, r, \varphi, \theta]^T$
$\hat{x}$ = estimate of state vector
y = measurement vector, $[p, q, r, n_z]^T$
δ_p = pilot input command vector, [collective, longitudinal, lateral, directional]^T

Matrices

A = open loop dynamics matrix
B = control distribution matrix
C = measurement distribution matrix, state vector
D = measurement distribution matrix, control vector

This is a revised version of a paper presented at the 43rd Annual Forum of the American Helicopter Society, St. Louis, Mo., May, 1987.

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$E(s)$ = multiplicative error matrix, $G_8^{-1}(s)\,G_{37}(s) - I$
$G(s)$ = open loop transfer matrix, $C(Is - A)^{-1}B$
$F(s)$ = full state loop transfer matrix, $K(Is - A)^{-1}B$
H = control mixer for pilot commands
$K(s)$ = compensator transfer matrix, $K(Is - A + BK + LC)^{-1}L$
K = feedback gain matrix
L = estimator gain matrix

Superscripts
T = transposed
-1 = inverse

Subscripts
8 = 8th order model
37 = 37th order model

Introduction

Recently a number of papers have appeared which discuss the application of various modern feedback control design techniques to helicopter flight control synthesis. In the past, classical single-input-single-output (SISO) frequency response techniques have been used to design control laws for helicopter flight control systems. However, since helicopter responses to control inputs are highly coupled, helicopter dynamics are characterized by multi-input-multi-output (MIMO) mathematical models and use of classical SISO techniques may require a great deal of time, consuming trial and error effort. Modern MIMO techniques are well suited to the design of control laws for helicopters, and numerous papers have appeared which describe such applications. These include linear quadratic regulator theory (Refs. 1-2), multivariable loop shaping (Ref. 3), model following (Ref. 4), optimal output feedback design (Ref. 5), and H^∞ techniques (Ref. 6). The advantages and disadvantages of some of these techniques are discussed in Ref. 5. The objective of this paper is to present the application of another technique, eigenstructure assignment, to the design of a helicopter flight control system.

Eigenstructure assignment is a technique for synthesis of feedback control laws which allows the designer to directly place closed loop eigenvalues and eigenvectors in specified configurations. These configurations are selected so that the closed loop response characteristics of the controlled helicopter satisfy handling qualities specifications. In this paper eigenstructure assignment is used to synthesize control laws for precise control of forward, side and vertical velocity and yaw rate for an attack helicopter in hover. It is desired to have pilot longitudinal stick commands correspond to forward velocity, lateral stick to side velocity, collective to vertical velocity, and pedal position to yaw rate. Coupling between longitudinal, lateral, vertical velocity and yaw modes is to be minimized. The closed loop bandwidth must be large enough to insure crisp response to pilot inputs and the closed loop dynamic response should be stable in the presence of errors in the design model, due to effects such as unmodeled dynamics, nonlinearities, and variations in parameters.

The rest of the paper is divided into five parts. First, the mathematical models used for both control system design and evaluation are discussed. Next, the performance requirements which the closed loop helicopter must meet are described. The design of the control system is outlined in the third major section and the evaluation and modification of the design is given in the fourth section. The last section consists of conclusions and suggestions for additional work.

Mathematical Models

The helicopter modeled in this study is a modern attack helicopter similar to the YAH-64 (Ref. 3). The control laws are designed using an eighth-order rigid body model and evaluated using a thirty-seventh-order model which includes actuator, rotor, sensor, and flexure dynamics. Main rotor collective pitch, longitudinal cyclic pitch, lateral cyclic pitch, and tail rotor collective pitch are the control inputs. Three body rate gyroscopes and an accelerometer which measures normal acceleration are used as sensors. The mathematical model used is semi-empirical and was developed for the hover flight condition. The linearized rigid body equations of motion are expressed in standard state variable form as

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx + Du \tag{2}$$

The A, B, C, and D matrices were obtained by numerical linearization of a nonlinear analytical model of the helicopter and are given in Table 1. The actuator/rotor and sensor flexural dynamics models used are given in Fig. 1 and were obtained from flight test data. Sensor outputs were measured and recorded for pilot inputs at various frequencies. The transfer functions shown in Fig. 1 were obtained by fitting assumed forms of the input and output transfer functions coupled with the model given in Eqs. (1) and (2) to the measured data and numerically adjusting time constants, damping factors, and natural frequencies until the frequency response of the model matched that of the helicopter. The validity of this type of model for flight control design was verified in Ref. 3 in which a flight control system was designed using a model similar to that given in Table 1 and Fig. 1. This control system was successfully flight tested.

The open loop eigenvalues and non-dimensional eigenvectors of the design model are given in Table 2. Non-dimensionalization of the state vector was achieved by dividing the linear velocities by 25 ft sec, the angular rates by 20 deg sec, and the angular displacements by 20 deg. These are the maximum values of the state variables expected during hover maneuvers. The control variables were non-dimensionalized by dividing by their maximum values of 9 deg collective, 15 deg longitudinal cyclic, 8.75 deg lateral cyclic, and 18.5 deg tail rotor collective pitch.

Examination of the eigenvectors and eigenvalues in Table 2 indicates significant coupling between lateral and longitudinal modes. Both forward and side velocities exhibit low frequency instabilities and as is discussed in the next section, the bandwidth in the pitch, vertical velocity, and yaw directions is not large enough to guarantee level 1 handling qualities. In addition to the modal coupling, examination of the control distribution matrix B in Table 1 reveals strong control coupling between lateral cyclic, longitudinal cyclic, and tail rotor collective. Only main rotor collective is relatively uncoupled. Time histories of open loop responses to a lateral cyclic step input are shown in Fig. 2. The response of the unaugmented helicopter is indicated by solid lines. It can be seen that the responses about all axes become very large due to the unstable eigenvalues and significant coupling which exists between modes.

The dynamic responses for the helicopter modeled in this study are typical of most high performance helicopters. Simulations and flight tests have shown that even experienced helicopter pilots are unable to accomplish relatively simple hover tasks in conditions of degraded visual cueing and/or divided attentions tasks with such typical helicopter dynamics (Refs. 7, 8). However flight tests with variable stability helicopters have shown that stability augmentation is an effective method for compensating for missing visual cues and for use in situations in which the pilot must devote a significant amount of time to tasks other than piloting.

Performance Requirements

Three response types have been proposed to quantify mission oriented rotorcraft handling quality requirements (Refs. 7-9).

Table 1 State and Gain Matrices

A =

−.0286	−.0637	.0205	.2290	7.9700	−.2570	.0000	−32.0000
.0779	−.2310	.0059	−8.2900	−1.0300	−1.6400	32.0000	.1640
.0046	−.0257	−.2610	−.3790	2.2500	2.1900	1.6000	−3.2800
.0079	−.0500	.0095	−2.70000	−.1340	−.6620	.0000	.0000
.0047	.0118	.0002	−.0092	−.7500	.0244	.0000	.0000
.0039	−.0049	.0008	−1.0500	.4130	−.4000	.0000	.0000
.0000	.0000	.0000	1.0000	−.0051	.1030	.0000	.0000
.0000	.0000	.0000	.0000	.9990	.0499	.0000	.0000

B =

.4350	.5760	−.1140	−.0009
−.1580	.1360	.4910	.2820
−4.2700	.0575	−.0250	.0012
−.0438	−.0600	.6470	.0800
.0072	−.1010	−.0900	−.0019
.0800	.0097	.2000	−.0455
.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000

C =

.0000	.0000	.0000	1.0000	.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000	1.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000	.0000	1.0000	.0000	.0000
−.0001	.0008	.0081	.0118	−.0699	−.0681	.0000	.0000

D =

.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000
.1327	−.0018	.0008	0.0000

H =

.6735	−.0785	−.0883	−.0044
−.1258	1.1040	−.2381	−.0255
−.0897	.0532	.2533	.1003
.7622	.3554	.8386	−.8169

L =

1.1421D+05	1.0690D+05	−1.9342D+04	6.0963D+04
1.0657D+05	1.0088D+05	−1.8388D+04	5.8262D+04
−9.7419D+05	−9.4997D+05	1.6548D+05	−5.1487D+05
3.5381D+03	2.7094D+03	−3.3272D+02	1.4736D+03
−1.6101D+03	−6.4545D+02	2.8823D+02	−1.1125D+03
−1.6727D+04	−1.6531D+04	3.0605D+03	−9.5008D+03
3.9652D+01	3.6522D+01	−7.3853D+00	2.3223D+01
4.8861D+00	6.1394D+00	−8.3640D−01	2.8294D+00

K =

−.0541	−.0145	−.1802	−.1214	.9174	−.5795	−1.9272	4.0079
.5974	−.1784	.0396	−.4143	−32.4650	−3.7509	−6.2656	−54.0280
.0628	−.0046	.0335	.3469	−.6961	4.1961	6.2384	−2.9799
.2346	.0013	−.1782	24.1517	−18.3387	−40.6498	21.6354	−20.3557

These are as follows:

1. Rate Command (RC)
2. Attitude Command with Attitude Hold (ACAH)
3. Translation Rate Command with Position Hold (TRCPH)

In RC systems, attitude must diverge from trim for at least 4 seconds following a step input command. In ACAH, a constant control input must produce a proportional angular displacement and must maintain this attitude in the presence of external disturbances. In TRCPH, constant control input must result in constant translational rate and the rotorcraft must hold position if the force on the cockpit controller is zero. TRCPH systems are preferred in nap of the earth maneuvers in fair to poor usable cue environments. In fair usable cue environments considerable concentration is required for the pilot to perceive pitch or roll attitude and lateral, longitudinal, or vertical translation rates (Ref. 7). The use of eigenstructure assignment for the design of RC and RCAH has been discussed in a previous paper (Ref. 10); therefore, this paper will concentrate on synthesis of a TRCPH using eigenstructure assignment.

Both classical (Ref. 11) and modern handling qualities literature (Refs. 7–9) indicate minimum bandwidths of 2 rad/sec in pitch, roll, and yaw rate and/or attitude and 0.25 to 0.75 rad/sec in vertical velocity are required for level 1 handling qualities. In addition, coupling between lateral, longitudinal, vertical velocity, and yaw modes should be minimized. Bandwidth requirements of 0.2 to 1 rad/sec have been postulated for TRCPH (Ref. 12).

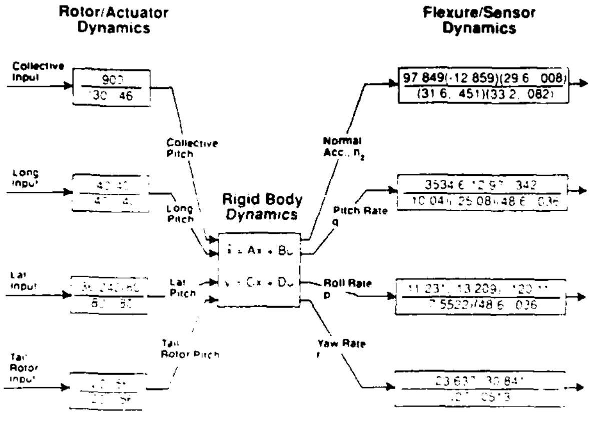

Fig. 1 Gain, pole, and zero locations for rotor/actuators and flexible modes: (λ) for real eigenvalues, (ω_n; ξ) for complex eigenvalues.

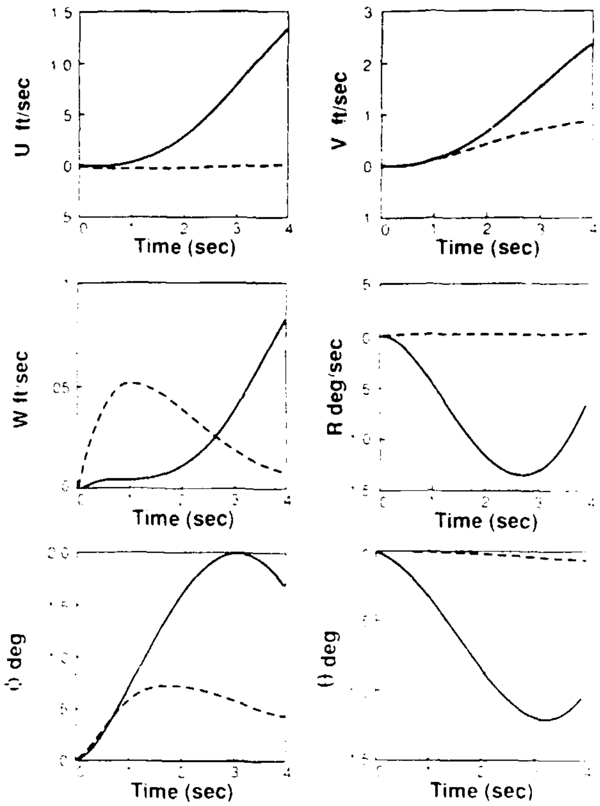

Fig. 2 Open loop and full state closed loop responses to a side velocity command: —— open loop, ---- full state feedback.

Feedback control systems must maintain stability in the presence of both uncertainties and errors in the mathematical models used for design and in variations in system parameters during actual operations. This is termed stability robustness. Stability robustness is usually specified indirectly in terms of gain and phase margin. Minimum gain margins of 6 db and minimum phase margins of 45° are typical stability criteria used for control law design.

Control Law Synthesis

The control law design process is performed in two stages for this study. First, a full state regulator is developed using eigenstructure assignment. Even though good decoupling of the closed loop eigenvectors is achieved, control coupling of the helicopter is so great that control command mixing is required. Both the feedback control law and the mixer are designed assuming full state feedback. The feedback control law cannot be implemented directly since all system states cannot be measured. Thus it is necessary to realize the control law by means of a state estimator in the feedback loop. The state estimator is synthesized using an eigenstructure assignment technique which results in recovery of the loop transfer properties of the full state regulator.

Full State Regulator

The feedback control law is a linear function of the state

$$u = -Kx \tag{3}$$

The feedback gain matrix K is selected to give a desired closed loop eigenvalue/eigenvector configuration. The theory for eigenstructure assignment by feedback control is given in

Table 2 Open-Loop Eigenvalues, Eighth-Order Model

Open-Loop Eigenvalues

1. −3.2610 + .0000i roll
2. −.9760 + .0000i pitch
3. .0820 ± .6296i side velocity
4. .1100 ± .5147i forward velocity
5. −.2588 ± .0428i yaw/vertical

Open-Loop Eigenvectors

	1.	2.	3.	4.	5.
u	−.0024	.3605	−.0710 + .3103i	−.0462 + .4723i	−.0244 − .1983i
v	−.0784	.0102	−.2171 + .1269i	−.0979 + .1297i	.0616 + .0923i
w	−.0012	.0525	.0040 + .0241i	.0082 + .0341i	−.0949 + .5468
p	−.8923	.0373	.1334 − .2352i	−.0066 − .1424i	.0850 + .0084i
q	.0268	−.6131	−.0234 + .2872i	.0666 + .2828i	.0488 + .0270i
r	−.3407	.3338	.3382 + .4070i	.4026 + .2892i	−.7740 − .1489i
φ	.2844	−.0767	−.2699 − .3026i	−.1982 − .1093i	−.0231 + .0234i
θ	−.0030	.6104	.4785 + .0727i	.5862 − .0431i	−.0257 − .0799i

Refs. 13–17. The details of the application of the theory to the problem described in this paper is given in Ref. 17; therefore, this section will be limited to a discussion of the philosophy of design and a presentaton of the results. The desired eigenvalues and eigenvectors for the final design are shown in Table 3. The desired eigenvalues were selected to satisfy the handling qualities specifications described above. The roll eigenvalue was selected to give a roll bandwidth of 3.5 rad/sec, a pitch bandwidth of 2.9 rad/sec, a yaw bandwidth of 3 rad/sec, and a vertical velocity bandwidth of 1 rad/sec. The eigenvalues associated with the forward and side velocities were selected to be complex with a natural frequency of 0.9 rad/sec and a damping factor of 0.9. This resulted in bandwidths of about 1 rad/sec for the forward and side velocities as well as the vertical velocity.

The desired eigenvectors are shown in Table 3. The first desired eigenvector associated with roll rate, p, is made equal to unity. Since the roll angle, ϕ, is the integral of the roll rate, the element associated with roll angle is the inverse of the roll eigenvalue. Also, since some side slip is inevitable when the helicopter is rolled, an arbitrary non-zero element is associated with side velocity. All other elements of this eigenvector are zero. The two complex eigenvectors associated with the side velocity complex roots, $-0.802 \pm 0.388i$, have side velocity elements equal to one and arbitrary non-zero values associated with roll rate and roll angle. All other elements are zero. Thus the lateral modes, roll and side slip, are decoupled from the longitudinal, vertical velocity and yaw rate modes. The desired eigenvectors associated with the pitch eigenvalue, -2.9, and the forward velocity eigenvectors associated with $-0.801 \pm 0.387i$, are selected in a similar manner to decouple the longitudinal modes from the lateral, vertical velocity and yaw rate modes. The eigenvector associated with the vertical velocity eigenvalue, -1.0, is selected so that all components are zero except the vertical velocity, which is unity. Similarly, all elements of the eigenvector associated with the yaw rate mode, -3.0, are selected as zeros, except for the element associated with the yaw rate which is chosen to be unity.

The attainable eigenvectors for a unity weighting on the squared error between all elements of the desired and attainable eigenvectors are shown in Table 3. Examination of the eigenvectors associated with roll indicates excellent decoupling between this and all modes, except the side velocity. The pitch mode is also decoupled from all modes except the forward velocity. Yaw and vertical velocity modes are also decoupled from the other modes. There is mild yaw coupling in the side velocity eigenvector and a more severe vertical velocity coupling in the forward velocity eigenvector. Responses were significantly

Table 3 Closed Loop Desired and Attainable Eigenvalues and Eigenvectors

Closed-Loop Desired Eigenvalues

1. −3.5000 + .0000i roll
2. −2.9000 + .0000i pitch
3. −.8020 ± .3880i side velocity
4. −.8010 ± .3870i forward velocity
5. −1.0000 + .0000i heave
6. −3.0000 + .0000i yaw

Closed-Loop Desired Eigenvectors

	1.	2.	3.	4.	5.	6.
u	.0000	x	.0000 + .0000i	1.0000 + .0000i	.0000	.0000
v	x	.0000	1.0000 + .0000i	.0000 + .0000i	.0000	.0000
w	.0000	.0000	.0000 + .0000i	.0000 + .0000i	1.0000	.0000
p	1.0000	.0000	x	.0000 + .0000i	.0000	.0000
q	.0000	1.0000	.0000 + .0000i	x	.0000	.0000
r	.0000	.0000	.0000 + .0000i	.0000 + .0000i	.0000	1.0000
φ	−.2857	.0000	x	.0000 + .0000i	.0000	.0000
θ	.0000	−.3448	.0000 + .0000i	x	.0000	.0000

UNITY WEIGHTING

Attainable Eigenvectors

u	.0024	−.12[illegible]	.0118 − .0307i	.4254 − .0000i	−.1003	−.0112
v	.0890	−.0235	.4745 − .0000i	.0131 + .0343i	−.0181	−.0366
w	0.0000	−.0293	−.0375 + .0192i	−.2279 + .0045i	.9939	−.0028
p	.9577	.0041	.3724 + .4431i	.0389 + .0346i	.0089	−.0065
q	.0001	.9357	.0092 + .0564i	−.4185 − .3999i	−.0290	−.0073
r	−.0002	−.0036	−.0282 + .0555i	−.0183 − .0082i	−.0014	.9986
φ	−.2736	.0003	−.5925 − .2726i	−.0571 − .0171i	−.0089	−.0321
θ	0.0000	−.3223	−.0368 − .0559i	.6197 + .1999i	.0290	−.0142

FINAL WEIGHTING

Attainable Eigenvectors

u	.0024	−.1381	.0118 − .0303i	.4143 + .0000i	−.1003	−.0112
v	.0890	−.0235	.4737 + .0000i	.0093 + .0332i	−.0181	−.0366
w	0.0000	−.0293	−.0376 + .0193i	−.0024 + .0000i	.9939	−.0028
p	.9577	.0041	.3720 + .4473i	.0408 + .0377i	.0089	−.0065
q	.0001	.9357	.0084 + .0581i	−.4350 − .4154i	−.0290	−.0073
r	−.0002	−.0036	−.0028 + .0056i	−.0192 − .0085i	−.0014	.9986
φ	−.2736	.0003	−.5943 − .2705i	−.0606 − .0194i	−.0089	−.0321
θ	0.0000	−.3223	−.0369 − .0549i	.6440 + .2075i	.0290	−.0142

decoupled, except in the two cases mentioned above. This was corrected by weighting the error between the desired and attainable vertical velocity element in the forward velocity eigenvector by a factor of 100. The yaw coupling, while not serious, was also reduced by increasing the weighting on the error in the yaw direction between the desired and attainable side velocity eigenvectors by a factor of 10. The attainable eigenvectors for this set of weighting terms are shown in Table 3. The resulting time histories for lateral step input for this design are shown in Fig. 2 by dashed lines. It can be seen that excellent decoupling has been attained. The forward velocity is nearly zero, as is the vertical velocity. Yaw and pitch angular velocities and pitch angle are also nearly zero. The roll angle overshoots its steady state value enough to produce a nearly constant side acceleration, which exists until the commanded side velocity is achieved. The roll angle then decreases until the side force is sufficient to counteract the steady state drag in the lateral direction, thereby maintaining constant side velocity. Similar results were obtained for longitudinal, collective, and directional commands. The gain matrix, K, for this design is given in Table 1.

In single-input-single-output (SISO) systems, stability robustness is measured by gain and phase margins obtained from Bode or Nyquist diagrams. For variations in the input gain and phase, the MIMO equivalent of these classical stability margins is the minimum singular value (MSV) of the return difference matrix $[I + K(s)G(s)]$ for $s = i\omega$ (Refs. 18–21). The MIMO gain and phase margins can be expressed as

$$\text{Gain Margin (GM)} = 20 \log(1/(1 \pm \text{MSV}))$$

$$\text{Phase Margin (PM)} = \pm \cos^{-1}(1 - (\text{MSV})^2/2) \qquad (4)$$

For a minimum singular value of unity, the gain margins are infinity and 6 db, with no change in phase at the input, and the phase margins are ± 60 deg with no change in gain at the input.

The minimum singular value of the return difference matrix for the full state design is plotted (solid line) versus frquency in Fig. 4. The minimum singular value of 0.67 occurs at 2.5 rad sec. From Eq. 4 this corresponds to gain margins of 9.63 db and 4.45 db and phase margins of ± 39 deg. Since the math model of the helicopter is felt to be reasonably accurate in this frequency range, it was felt that these margins were sufficiently large to guarantee stability. This proved not to be the case, however.

Control Mixer Design

Even with the modal decoupling in the closed loop system, the control coupling resulted in excessively coupled responses

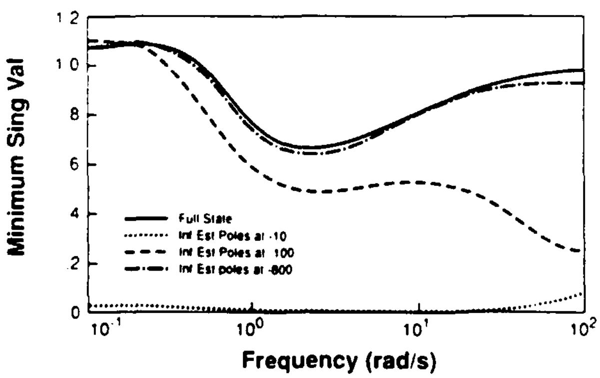

Fig. 3 Minimum singular values vs. frequency for full state feedback and various estimator pole locations.

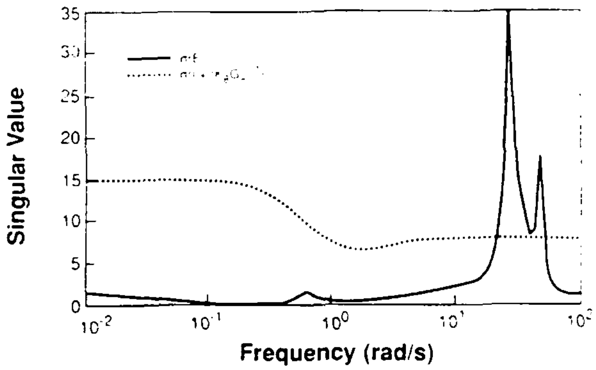

Fig. 4 Minimum singular values of closed loop transfer function and maximum singular values of multiplicative error matrix vs. frequency.

and a fixed gain control mixer was required to convert pilot inputs to control inputs (note the control mixer described below was used in obtaining the results in Fig. 2). The control mixer was designed by calculating the control u required to produce a given steady state response. The details of this design are given in Ref. 17. The resulting control mixer gain matrix, H, is given in Table 1. The closed loop system equations are now

$$\dot{x} = [A - BK]x + BH\delta_p \qquad (5)$$

where δ_p represents pilot command.

Compensator Design

The control law developed above requires knowledge of the complete state vector. Since only measurements of roll, pitch, and yaw rates and normal acceleration are available, a state estimator is required in the feedback loop. The control is

$$u = -K\hat{x} \qquad (6)$$

where $\hat{x}$ is the estimate of the state given by the state estimator

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x} - Du) \qquad (7)$$

It is well known that an estimator such as given by Eq. 7 may not recover the stability margins of the full state controller (Refs. 18–21). In fact, the stability margins may be very poor, even though those of the full state controller may be excellent. As described in Ref. 22, an estimator which recovers the full state loop transfer stability properties can be designed by appropriate selection of the estimator gain matrix L such that: (1) j of the closed loop eigenvalues of $A - LC$ approach the finite transmission zeros of the plant, $G(s)$; (2) the remaining $n - j$ closed loop eigenvalues approach infinity, and (3) the left closed loop eigenvectors of $A - LC$ associated with the finite eigenvalues approaches the left zero direction of the finite transmission zeros. This results in a controller in which the full state loop transfer matrix $F(s)$ is approximated by the compensator loop transfer matrix $K(s)G(s)$ up to the frequency of the infinity poles.

The finite transmission zeros of $G(s)$ were calculated as

$$z_1 = 0.0$$

$$z_2 = 0.0$$

$$z_3 = -0.02147$$

$$z_4 = -0.11224$$

In designing the estimator, the gain matrix L was selected to place the two of the estimator poles at z_1 and z_4 and two of the estimator poles at -0.01 and -0.012 (these approximate the transmission zeros at zero). Initially the remaining estimator poles were placed at -10, -12, and $-5 \pm 8.666i$ (these approximate the transmission zeros at infinity). The resulting singular value plot is shown in Fig. 4. Stability margins are almost nonexistent. The "infinity" pole locations were increased by a factor of ten to -100, -120, $-50 \pm 86.666i$. The singular value improves substantially but is still unsatisfactory at high frequencies. Finally, the "infinity" pole locations were increased by another factor of eight to -800, -960, $-400 \pm 693.3i$. This pole configuration resulted in essentially full recovery of the full state singular values over the entire frequency range. The resulting estimator gain matrix, L, is given in Table 3.

Evaluation

The design developed above using the eighth-order model was evaluated using the 37th order model described in Fig. 1. The MIMO generalized gain and phase margins of 9.63 and -4.45 db and ± 39 degrees were felt to be adequate to provide stability in the presence of unmodeled dynamics. However, the closed loop 37th order system exhibited an instability near 30 rad/s due to coupling between the main rotor collective/actuator dynamics and the sensor flexural dynamics as measured by the normal accelerometer. Instabilities resulting from rotor and sensor dynamics have been noted by Chen and Hindson (Ref. 23) and Hall and Bryson (Ref. 1). To better understand the source of the instability the multiplicative error matrix, $E(s)$, between the 8th order model and the 37th order model was computed at various frequencies. This matrix is defined (Ref. 18) as

$$G_{37}(s) = G_8(s)(I + E(s)) \tag{8}$$

To be assured stability in the face of modeling errors $E(s)$, it is known (Ref. 18) that at all frequencies

$$\underline{\sigma}(I + (K_8G_8)^{-1}) \geq \bar{\sigma}(E) \tag{9}$$

A plot of these two functions is shown in Fig. 5. The large error peak near 30 rad/s crosses the minimum singular value curve verifying the source of the instability. To alleviate this problem one might increase the order of the design model to include dynamics in the 30 rad/s range and completely redesign the compensator. However, a filter on the accelerometer signal which notched out the set of complex poles at 30 rad/s and rolled off at 200 rad/s was implemented and was found to eliminate the instability.

Transient responses to pilot commands for the 37th order model with the notch filter are given in Fig. 5. Figure 5 illustrates a pure side velocity maneuver resulting from step pilot lateral input, of 1 ft/sec. In this case longitudinal, vertical velocity, and yaw responses are minimal. In general, the responses in Fig. 6 are very close to the full state responses of Fig. 2 with the exception of small lags due to actuator dynamics. The control deflections required to achieve these responses were not large (Ref. 17).

Closed loop transfer functions between pilot commands and system outputs are shown in Fig. 6. These figures demonstrate (as the transient responses did) that the closed loop system is now characterized by simple decoupled first order responses over the desired bandwidth range. The vertical velocity, forward, and sideslip transfer functions are flat until about 0.8 rad/sec and then begin to roll off. The yaw transfer function is flat until about 1 rad/sec then rolls off. If necessary the bandwidth on these transfer functions could have been increased by increasing the natural frequency of the complex closed loop eigenvalues.

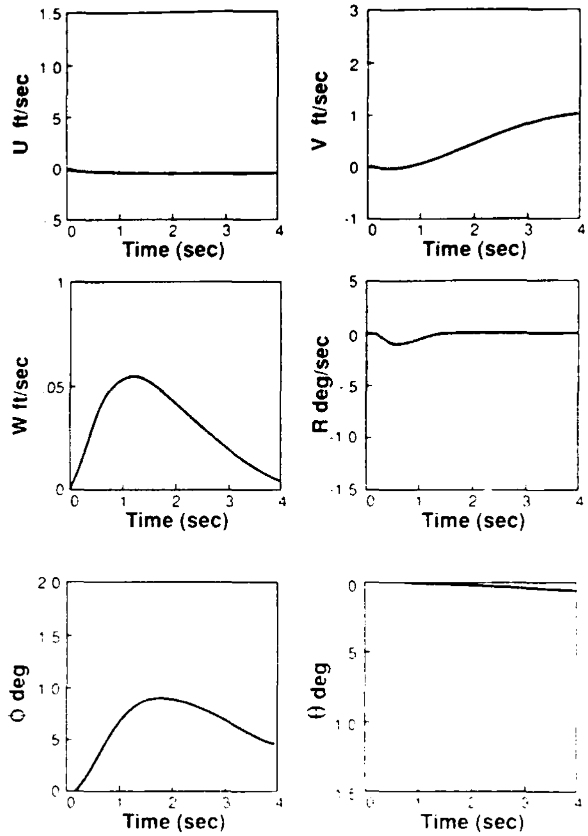

Fig. 5 Closed loop time responses to a side velocity command—37th order model.

Conclusions

The eigenstructure design techniques described in this paper provide a useful method for the design of control laws for helicopter flight control systems for precision hover tasks. Use

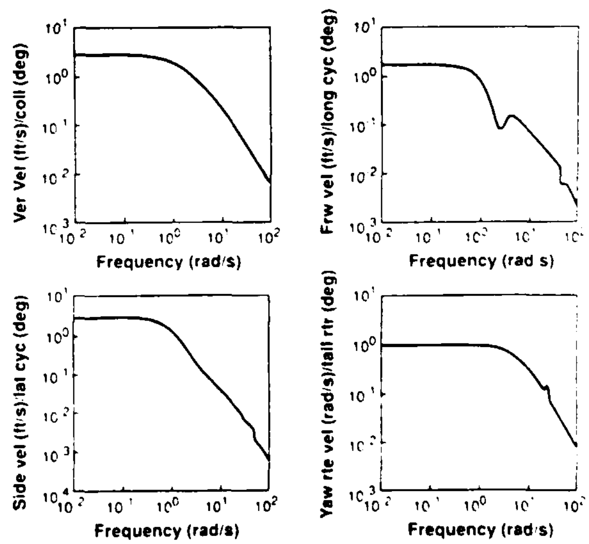

Fig. 6 Bode plots of closed loop transfer functions between input commands and system outputs.

of MIMO gain and phase margins based on minimum singular values of the loop gain did not predict an instability due to actuator/rotor and sensor/flexural dynamics in the collective control loop. This instability was eliminated by filtering the accelerometer output through a notch filter. The use of an error model derived from an estimate of the dynamics neglected in the design model did predict the instability and it is recommended that this approach to evaluating stability margins be used in the future. The state estimator in the feedback loop required high gains. This might cause problems in actual implementation unless the sensor outputs are filtered. Also, the effects of digital implementation of the control laws might cause difficulties if the sampling and computational rates are sufficiently slow, but this is dependent on hardware considerations and is not addressed in this paper.

Acknowledgment

This research was supported by the Army Research Office under contract DAAL03-86-K-0056. Mr. Mark Ekblad and Mr. Joseph Anderson, Research Assistants at the University of Minnesota, provided invaluable assistance. All computing and graphics presented here were performed on Apple Macintosh computers using *MAControl*™, a control system design package developed by Professor Bradley Liebst and Mr. Jerry Farm of the University of Minnesota.

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